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A defence against the next convexity crunch

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Crédit Agricole CIB rates traders describe a new way of hedging the risk of bond convexity

Long-term bonds are riding a wave of popularity, especially for high-quality issuers such as sovereigns, supranationals and agencies. Austria's 100-year government bond, for instance, was nearly nine times oversubscribed on launch last year.

For banks, the appeal of these bonds is the slightly higher yields, quasi risk-free nature, and ability to act as a high-quality liquid asset. Asset managers are drawn to longer-dated assets to match the lengthening duration of their liabilities. And the increased demand raises the prices of these bonds in a way that's attractive to more speculative players.

But as more money moves to the end of the bond curve in Europe, investors face greater exposure to a phenomenon that becomes more influential in a low-rate environment – convexity. In other words, with rates where they are, any movement in expectations has an outsized impact on the duration and values of these long-dated bonds.

This convexity creates issues for certain market participants. An asset-liability manager, for example, typically targets a specific duration gap between assets and liabilities. But when rates expectations move around, so does the size of this gap, an effect known as a convexity gap.

The convexity gap can be hedged using combinations of bonds, swaps or swaptions, but these introduce more duration exposure.

A purer hedge can be something called a convexity swap. Using a new formula explained below to effectively derive the price of convexity at a given point of the curve, we believe market participants can cleanly hedge their convexity risks without the ensuing duration risks.

What is convexity?

To define convexity, you must first define duration. In 2007, when interest rates were at 5%, an investor had to pay \$20 to guarantee \$1 per year indefinitely. For a rate of 1% an investor would have to pay \$100 to guarantee \$1 per year indefinitely. When rates are negative, there are no guaranteed returns.

This is expressed by the equation that defines duration/perpetual:

$$\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\cdots+\frac{1}{(1+r)^n}+\cdots=\frac{1}{r}\quad (r>0)$$

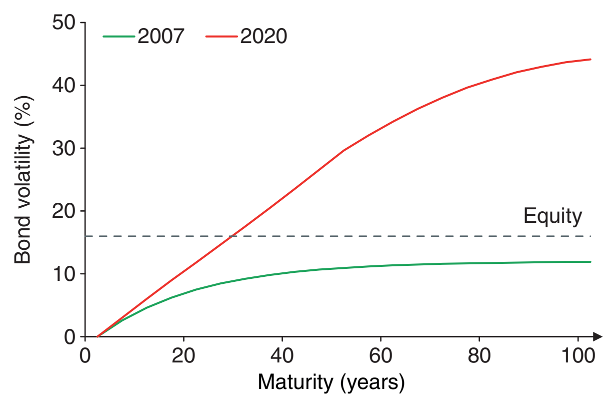
For the purposes of this article, we assume interest rates are greater than zero.

The value needed to secure \$1 over a given period of time is the duration and/or annuity, which depends on the level of interest rates and is interpreted as the average life of a bond; duration also represents the sensitivity of bond prices to changes in interest rates.

The duration of a bond depends on interest rate levels: the lower the interest rates the higher the duration. Convexity describes how a change in the level of interest rates affects duration.

When rates are low, the price of a bond is more sensitive to changes in interest rates, which makes the duration more important. The convexity of a bond is an increasing function of its maturity, the longer the maturity the higher the convexity.

1 Bond volatility



The perpetual duration equation shows the importance of this convexity in the case of long-term assets and its explosive nature when rates are close to zero. The direct consequence of lower interest rates on long-term bonds is an increase in their volatility.

While implied volatility of interest rates has remained similar over the past 15 years, the implied volatility of long-term bonds has risen sharply. For example, the implied volatility of interest rates was 9% 15 years ago, which is 5 basis points per day or 80bp per year, and has remained fairly constant through that time. The implied volatility of a 30-year bond, however, is nearly double at 17% today (figure 1).

The convexity effect has therefore mechanically increased the volatility of long-term bonds, to the point where it is higher than that of stocks. The volatility of the Euro Stoxx 50 index is 16%, for example. This has led to a situation where an asset has increased its volatility while reducing its return, which may seem counterintuitive.

Fundamental equation of convexity

In general, the price of a bond is described as a function of three factors: interest rate expectations, risk premium and convexity:

$$\text{bond price}=\underbrace{\text{Rates Anticipation}}_{\text{short term}}+\underbrace{\text{Risk Premia}}_{\text{medium term}}+\underbrace{\text{Convexity}}_{\text{long term}}$$

Any bond is influenced by these three factors; however, depending on the maturity of the bond, one of the factors dominates. Rate expectations are the dominant factor for short-term bonds (<5y), risk premium is the dominant factor for medium-term bonds (5y–20y), convexity is the dominant factor for long-term bonds (>20y).

The risk premium factor tends to lower the price of bonds/increase the yield, while the convexity factor varies in the opposite direction; it tends to increase the price of bonds/lower the yield.

To formalise the convexity of a bond, it is sufficient, on the one hand, to describe the variation in value of the bond between two rate levels, and on the other hand, to determine the difference between the value of the bond and the value of the tangent at the rate level considered.

To do this, we consider a bond at par (price = 100) (figure 2):

$$\begin{aligned}
\text{BondPrice}(r) &= \sum_{i=1}^{n} \frac{r_0}{(1+r)^i} + \frac{100}{(1+r)^n} \\
&= \sum_{i=1}^{n} \frac{r_0}{(1+r)^i} + 100 - \sum_{i=1}^{n} \frac{r}{(1+r)^i} \\
&= 100 + \sum_{i=1}^{n} \frac{r_0 - r}{(1+r)^i} \\
\text{Duration}(r) &= \sum_{i=1}^{n} \frac{1}{(1+r)^i} \\
\text{BondPrice}(r) &= \underbrace{100}_{\text{Cash}} + \underbrace{\text{Duration}(r) \times (r_0 - r)}_{\text{Swap}} \\
\text{Tangent}_0(r) &= 1 + \text{Duration}(r_0) \times (r_0 - r) \\
\text{Convexity} &= \text{BondPrice}(r) - \text{Tangent}_0(r) \\
&= 1 + \text{Duration}(r) \times (r_0 - r) \\
&\quad - 1 - \text{Duration}(r_0) \times (r_0 - r) \\
&= (\text{Duration}(r) - \text{Duration}(r_0)) \times (r_0 - r) \\
&= \Delta\,\text{Duration} \times \Delta\,\text{Rate}
\end{aligned}$$

The fundamental equation for the price change of a par bond:

$$\Delta\,\text{BondPrice} = \text{Duration}\,\Delta r + \Delta\,\text{Duration}\,\Delta r$$

The fundamental equation of convexity is written:

$$\text{Convexity} = \Delta\,\text{Duration}\,\Delta r$$

The change in the price of a bond is broken down into two parts: a linear part that depends on the duration and a non-linear part that depends on the change in duration.

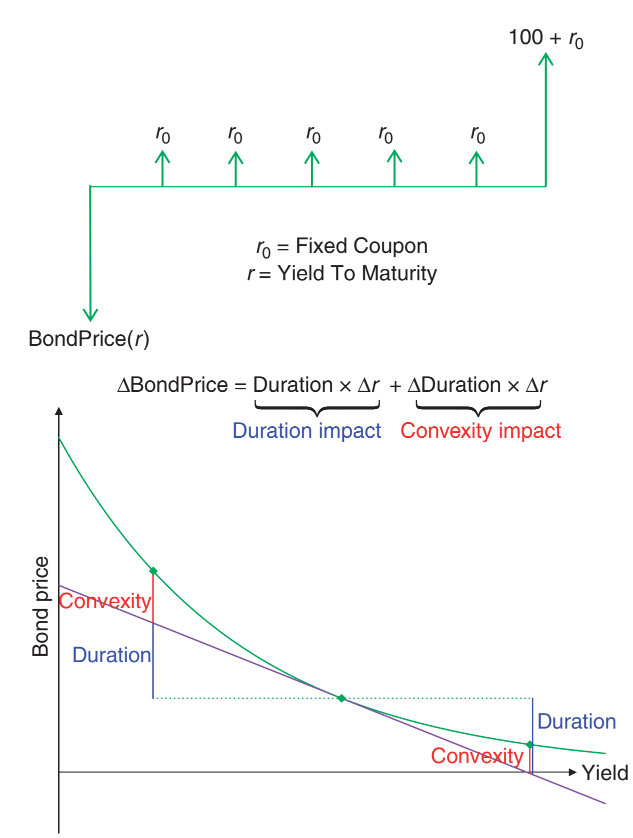
Convexity represents the non-linear change in the price of the bond. The convexity impact corresponds to the product of the variation in duration and the variation in rates. This product is fairly intuitive: the more rates vary, the more the duration varies, and the greater the impact of convexity.

The value of this convexity is contained and implicit in the price of the bond. The more the market anticipates volatility, the greater the rise in convexity will be, which pushes the price of the bond up and therefore mechanically the yield down. Investors will give up part of the yield to take advantage of or protect themselves from market variations.

As an example, let's consider a bond with a maturity of 30y (without credit risk) whose value is 100% (bond at par) with a yield of 1%. Let's suppose that the market anticipates a volatility of 1% with two possible states of the world that are equally probable: 30y at 0% and 30y at 2%. In the first case the value of the bond will be 130%, in the second case 77.6%. The expected gain and therefore the market value on average would be 103.8%, ie, an implicit yield of 0.85%.

This additional cost of 3.8% corresponds to the expected gain generated by convexity: 4% gain if rates fall by 1% and 3.6% gain if rates rise by 1%.

2 A bond at par



Thus, the more the market anticipates volatility, the more the value of long-term bonds, which benefit from strong convexity, will increase. The direct consequence is the flattening of the yield curve, especially in the long end.

To give an intuition of the behaviour of convexity, we can use a Taylor development and apply it to the case of perpetual duration in the case of a positive rate:

$$\Delta\,\text{PerpetualDuration}\,\Delta r = \frac{(\Delta r)^2}{r^2} - \frac{(\Delta r)^3}{r^3} + \frac{(\Delta r)^4}{r^4} + \cdots$$

In general, we have:

$$\begin{aligned}
&\Delta\,\text{Duration}\,\Delta r \\
&\quad \propto \text{Duration}^2(\Delta r)^2 - \text{Duration}^3(\Delta r)^3 + \text{Duration}^4(\Delta r)^4 + \cdots
\end{aligned}$$

By looking at the first term of the equation, we can approximately determine the expectation of the convexity:

$$\begin{gathered}
\text{Convexity price} = E[\Delta\,\text{Duration}\,\Delta r] \approx E[(\text{Duration}\,\Delta r)^2] \\
E[(\text{Duration}\,\Delta r)^2] = E[(1 + \text{Duration}\,\Delta r)^2] - E[1 + \text{Duration}\,\Delta r]^2 \\
\text{Convexity price} \approx \text{Variance(Bond)}
\end{gathered}$$

We observe a strong link between convexity and duration: the more duration increases, the more convexity accelerates. Consequently, the convexity of bonds decreases over time faster than the duration.

We can also observe that convexity value is proportional to the variance. The variance underlines the strong dependence on the variation of rate – even more so when the rate variation is extreme.

We deduce the volatility of the bond given by:

$$\text{Volatility(Bond)} \approx E[\sqrt{\Delta \, \text{Duration} \, \Delta r}]$$

Convexity: a global phenomenon and a challenge for ALM

The objective of asset-liability management is to match assets to liabilities, and thus hedge the various risks. Just as assets are often composed of bonds, liabilities, which correspond to a commitment to investors, are also modelled as bonds. The first-order interest rate risk is the difference in sensitivity to interest rates between assets and liabilities – in other words, the difference in duration between liabilities and assets.

This difference is called the 'duration gap'. The gap evolves according to the level of interest rates, so the management of this gap impacts investment policies.

The duration gap is said to be open as soon as the duration of assets and liabilities is different. If the duration of the liabilities is greater than that of the assets, the gap is positive; if the duration of the liabilities is lower than that of the assets, the gap is negative.

This gap creates interest rate risk, and dynamic management of this gap consists of reducing this gap to zero or maintaining it at a desired level. Let's take an example of an institution with a long-term liability of 20 years and a shorter-term asset of 10 years. The institution has an open and a positive duration gap.

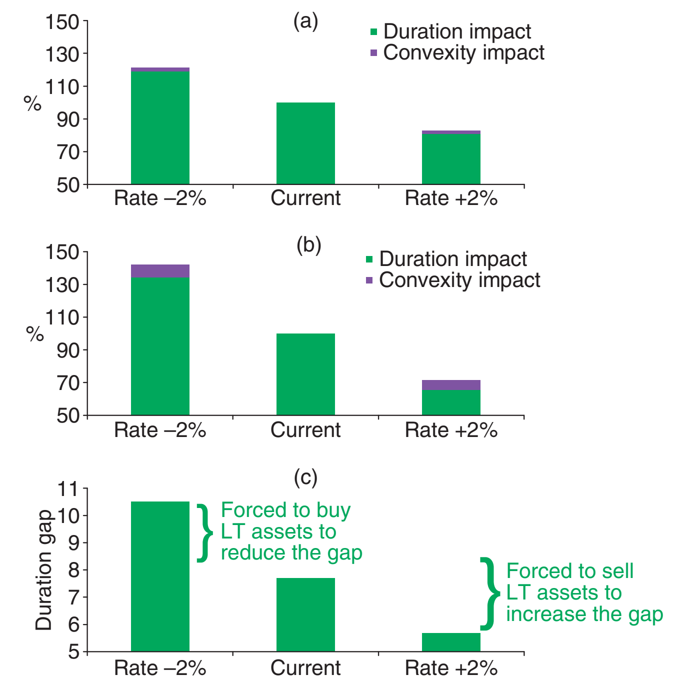
The 'convexity gap' corresponds to the sensitivity of the duration gap to interest rates. For a positive duration gap, if interest rates fall the duration gap will widen and become more and more important. This will force the institution to hedge the drift/variation of this gap and thus to buy assets while rates fall. Equally, to maintain the level of gap the institution will have to sell assets if rates go up.

This drift is explained by the fact that the long-term duration (liabilities) drifts/changes faster as a function of rates than the short-term duration (assets). Thus the economic cost of negative convexity is linked to the rebalancing frequency, and materialised by the round-trip costs of closing/maintaining the duration gap after each change in the level of interest rates (figure 3).

Another example in the retail banking world in the US and Europe is that of residential mortgage-backed securities. In simple terms, on the one hand, banks receive floating deposits from homeowners, the remuneration depends on market rates and admits a 0% guarantee. These deposits constitute the bank's liabilities. On the other hand, banks grant real estate loans which constitute their resources – or assets.

Assets and liabilities admit a duration which depends on the modelling of the behaviour of homeowners, starting from a given level of rates and a given duration spread. If rates decrease, the proportion of homeowners to repay their mortgage loan will increase, which reduces the duration of the assets, while the deposits they make are close to the capital guarantee which implies a lengthening of the duration. Thus, a fall in interest rates will negatively increase the gap and lead banks to buy long assets while their price rises to cover its variation. This phenomenon is symmetrical to the rise in interest rates: homeowners repay their loans less and deposits will move to higher yielding assets if they are not remunerated at the market rate, reducing the gap, so that an asset will have to be sold in a bear market. Thus, the management of the duration gap highlights the phenomenon of negative convexity.

3 (a) 10-year bond (asset), (b) 20-year bond (liability) and (c) duration gap



The situation of an insurer with a capital guarantee and a right of exit granted to the insured without penalty (lapse-surrender risk) is similar to the case of the bank.

In summary, a duration gap with liabilities longer than assets, as well as the options granted to homeowners/savers, induces negative convexity in the balance sheet of financial institutions.

Moreover, the negative convexity creates a self-reinforcing dynamic, and this convexity management implies that the demand for bonds is convex to the bond price.

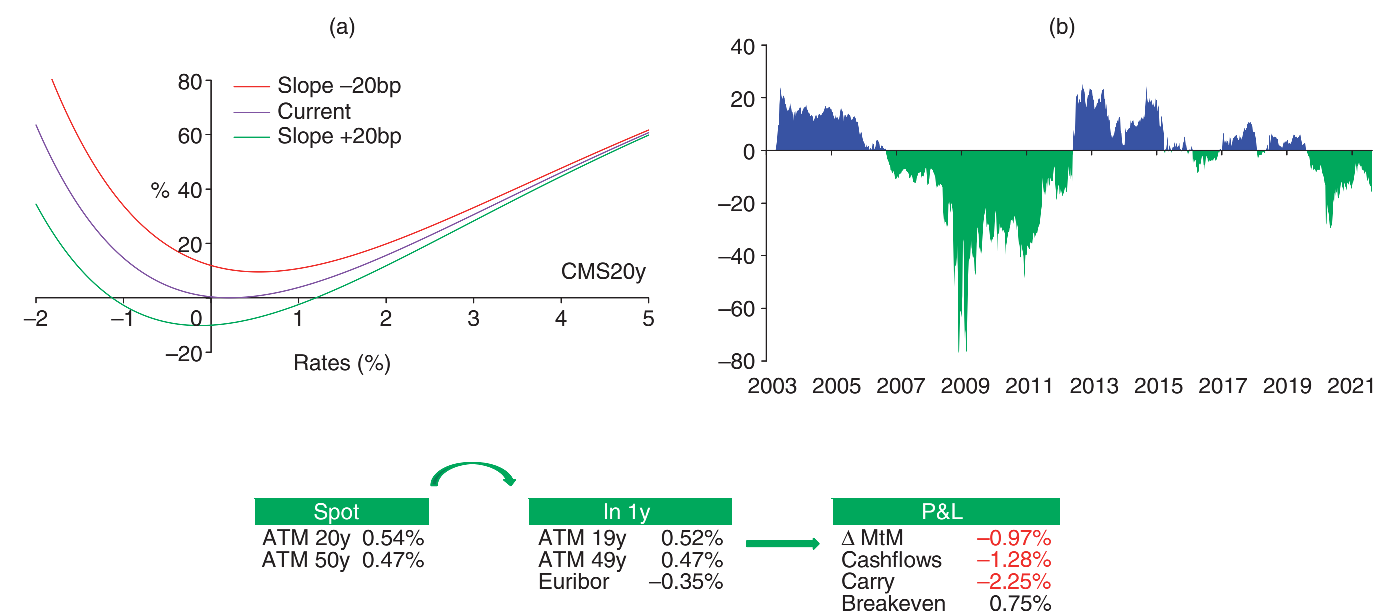
Convexity and shape of the curve

As we have seen, there is a strong link between convexity, the shape of the yield curve and the market's expected volatility. The expected volatility has an impact on the shape of the yield curve. The more volatility the market anticipates, the more the shape of the curve is concave and sloped down over the long term, and the more expensive it is to index/hedge against convexity.

The two usual convexity strategies are the flattener and the fly. The flattener consists of buying a long-term bond and selling a shorter-term bond for an amount that locally offsets the sensitivity to a parallel shift in the yield curve, or entering into a long-term receiver swap and a shorter-term payer weighted in the same way. Thus indexing positively to convexity and taking advantage of parallel movements in the yield curve implies a flattening position of the curve and a carry position:

$$\begin{aligned}\text{Flattener}(r_{\text{LT}}, r_{\text{CT}}) &= \text{Duration}_{\text{LT}}(r_{\text{LT}})(r^0_{\text{LT}} - r_{\text{LT}}) \\ &\quad - \Delta(r^0_{\text{LT}}, r^0_{\text{CT}}) \cdot \text{Duration}_{\text{CT}}(r_{\text{CT}})(r^0_{\text{CT}} - r_{\text{CT}})\end{aligned}$$

4 (a) MtM of long 50y/short 20y duration neutral and (b) slope 50y–20y



with

$$\Delta(r^0_{\mathrm{LT}}, r^0_{\mathrm{CT}}) = \frac{\mathrm{Duration}_{\mathrm{LT}}(r^0_{\mathrm{LT}})}{\mathrm{Duration}_{\mathrm{CT}}(r^0_{\mathrm{CT}})}$$

The flattener strategy is convex with respect to parallel rate movements and has a unique minimum at the point $(r^0_{\mathrm{LT}}, r^0_{\mathrm{CT}})$.

Consider the purchase of \$100m of a 50-year bond at par against the sale of \$235m of a 20y bond at par, or entering a 50y at-the-money receiver swap for \$100m against entering a 20y at-the-money payer swap for \$235m.

The 20y is chosen because it is generally the highest rate on the swap curve and is influenced by the risk premium factor, and the 50y is chosen because it is the longest 'LCH-cleared' rate and therefore benefits from the maximum convexity (figure 4).

The cost of convexity is determined according to the cost of carry – in other words, the value of the strategy (roll down) if market conditions are unchanged in a year's time. This carry cost depends on the level of the slope: the more the curve is inverted, the higher the carry cost. This cost of carry also defines the rate change needed to absorb this cost. In our example, the carry cost for the first year is 2.25%; this cost will be absorbed if the rates vary by 0.75% or more.

The flattener is a flattening strategy, the value of the strategy increases/decreases if the spread 50y–20y decreases/increases. Finally, it can be observed that in turbulent market situations the 50y–20y flattens, thus materialising the anticipation of volatility and reinforcing the price of convexity.

A second type of convexity strategy is the barbell/fly, which is a long position in a short-term and long-term bond and the sale of a medium-term bond for an amount that locally offsets the sensitivity to a parallel shift in the yield curve. This can also be replicated by payer and receiver swaps.

The barbell/fly offers a position on the curvature/twist of the yield curve. The more concave the yield curve, the greater the expected volatility.

The convexity swap: a bridge between the cash market and the derivatives market

The convexity swap is a swap in which a premium is exchanged for a payment that reproduces the convexity of a long-term bond. The principle is to apply the fundamental convexity equation to a swap rate (eg, CMS30y) as its value is published on the official page for the Ice swap rate (figure 5):

$$\text{Convexity payoff} = \Delta\,\text{Duration}\,\Delta r$$
$$\Delta\,\text{Duration} = \text{Duration}_{30}(r) - \text{Duration}_{30}(r_0)$$
$$\Delta r = r_0 - r$$
$$r = \text{CMS30y}, \quad r_0 = \text{CMS30y}_0$$
$$\text{Duration}_{30}(x) = \sum_{i=1}^{30} \frac{1}{(1+x)^i}$$

- **Application 1 (rates decrease):**

$$\text{CMS30y}_{\text{initial}} = 1\%, \qquad \text{CMS30y}_{\text{end}} = 0\%$$
$$\text{Duration}_{30}(\text{CMS30y}_{\text{initial}}) = 25.8$$
$$\text{Duration}_{30}(\text{CMS30y}_{\text{end}}) = 30$$
$$\implies \Delta\,\text{Duration}\,\Delta r = 4.2\%$$

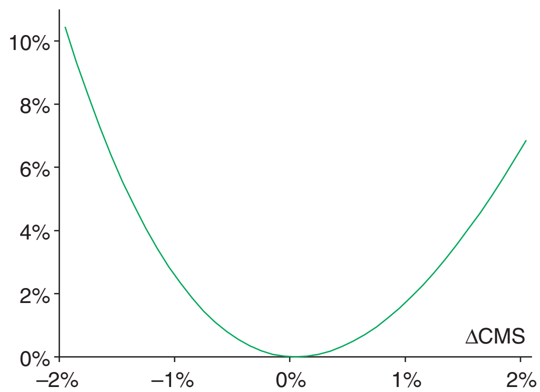
- **Application 2 (rates increase):**

$$\text{CMS30y}_{\text{initial}} = 1\%, \qquad \text{CMS30y}_{\text{end}} = 2\%$$
$$\text{Duration}_{30}(\text{CMS30y}_{\text{initial}}) = 25.8\%$$
$$\text{Duration}_{30}(\text{CMS30y}_{\text{end}}) = 22.4$$
$$\implies \Delta\,\text{Duration}\,\Delta r = 3.4\%$$

The convexity swap makes it possible to hedge or index the convexity of a long-term bond without taking any exposure to the duration.

The payoff of the convexity swap thus synthetically replicates the convexity of a long-term bond, and pays the value of this convexity over a given period.

5 Convexity swap payoff



A fundamental property of the convexity swap is that it keeps the convexity constant over time, whereas bonds lose their convexity quadratically over time.

To intuit the behaviour of the convexity swap one can approximate it as follows:

$$\text{Convexity30y payoff} \cong -\frac{\partial\,\text{Duration}}{\partial r}(\Delta r)^2$$

For the 30y in the interval [−2%, 7%], we have the following approximation:

$$-\frac{\partial\,\text{Duration}}{\partial r} \cong \tfrac{1}{2}\,\text{Duration}^2$$

Thus we can deduce a robust estimate price of the value of the convexity 30y:

$$\text{Convexity30y price} = E[\text{Convexity30y payoff}] \cong \tfrac{1}{2}\,\text{Duration}^2\,\sigma^2 T$$

For example, for a 30y rate at 1% and an implicit volatility of 70bp per year, we can easily deduce the value of the convexity at 1 year which is 1.63% ($\frac{1}{2}(25.8^2) \times 0.70\%^2$), which implies that the premium will be absorbed if the rates have a variation greater than or equal to 0.70%. It becomes easy to compare the price of the convexity implied by the swaption market and that implied by the swap rate curve.

The term $\text{Duration}^2\,\sigma^2$ is very useful to understand the value of convexity and its behaviour. Indeed, we observe that the value of convexity is convex in duration and implied volatility (deduced from the swaptions market), which implies that low interest rates and high volatility of rates tend to significantly increase the value of convexity.

Thus, a strategy of hedging convexity in rates, ie, through adjustments via swaps, will be all the more dynamic and important in the context of low rates. This implies that a hedging strategy based on one or a few swaptions will require a rebalancing that is all the more dynamic in a context of low rates and high flight.

Thanks to this approximation we can deduce the Greeks written as a function of the duration:

$$\text{Convexity30y Greeks} \rightarrow \theta \cong -\tfrac{1}{2}\,\text{Duration}^2\,\sigma^2$$
$$\gamma \cong \text{Duration}^2, \qquad \vartheta \cong \text{Duration}^2\,\sigma T$$

We have seen that when the market anticipates volatility, the hunt for convexity for opportunity or hedging purposes implies a flattening of the curves and thus a mechanical increase in the price of convexity. In the swaptions market, this implies buying swaptions of short-term expiry and long underlying (eg, 1y30y).

The magnitude of the phenomenon can be characterised by the Vanna = $\partial\text{Vega}/\partial r$, which is well known to swaption market participants:

$$\text{Convexity30y Vanna} \cong -\text{Duration}^3\,\sigma T$$

We observe a negative vanna and highly convex duration, which has the potential to create strong instabilities and erratic market behaviour, as was the case during the 2008, …, 2020 crises. This formula shows that when rates fall, the need to purchase volatility or protection increases drastically.

Convexity is a phenomenon that captures sudden and large movements, hence the dependence on short-term options, and the formulas show that it is a stable phenomenon in rate and time. Hedging with, say, out-of-the-money swaptions is a challenge in that if rates move, the value of these out-of-the-money options will have to be locked in and others bought back to maintain the level of the convexity hedge constant. When time passes, the out-of-the-money options lose their convexity contribution very quickly, which implies having to roll the hedge. The convexity swap benefits from stability in time and space with behaviour that depends on the duration at all orders.

The formulation of the convexity of a bond is replicated by a continuum of cash-settled swaptions at-the-money and out-of-the-money and is expressed as follows:

$$\begin{aligned}\text{Convexity30y payoff} = &\int_{-\infty}^{r_0} g''(k)\,\text{Duration}(r)(k-r)_+\,\mathrm{d}k\\ &+\int_{r_0}^{+\infty} g''(k)\,\text{Duration}(r)(r-k)_+\,\mathrm{d}k\end{aligned}$$

with:

$$g(x) = \left(1 - \frac{\text{Duration}(r_0)}{\text{Duration}(r)}\right)(r_0 - r)$$

Conclusion

In conclusion, convexity in the bond world is a major global phenomenon, more sensitive in low interest rate environments and more impactful when markets are volatile. This convexity implies in the current context a change in the bond world with the price of long-term bonds becoming more volatile when interest rates are low.

The increase in the volume of long-term bond issues and therefore in the duration in the bond market reinforces the potential for volatility and therefore the risks of convexity crises.

The introduction of the convexity swap establishes a bridge between the cash market and swaptions market and helps reproduce the implicit convexity of long-term bonds in the swaption market.

The analysis of the convexity swap shows that the dynamics and the behaviour of the convexity depend on the duration. We observe that the convexity behaves as the square of the duration.

Convexity is an important driver for many financial institutions, both for risk management objectives and the search for opportunities. ■

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